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ABSTRACT

This pamphlet introduces the student to the basic ideas and procedures of transformational geometry through a series of worksheets. After developing intuitively the idea of rigid motion, vector diagrams are introduced, and translations are discussed in some detail. (SD)

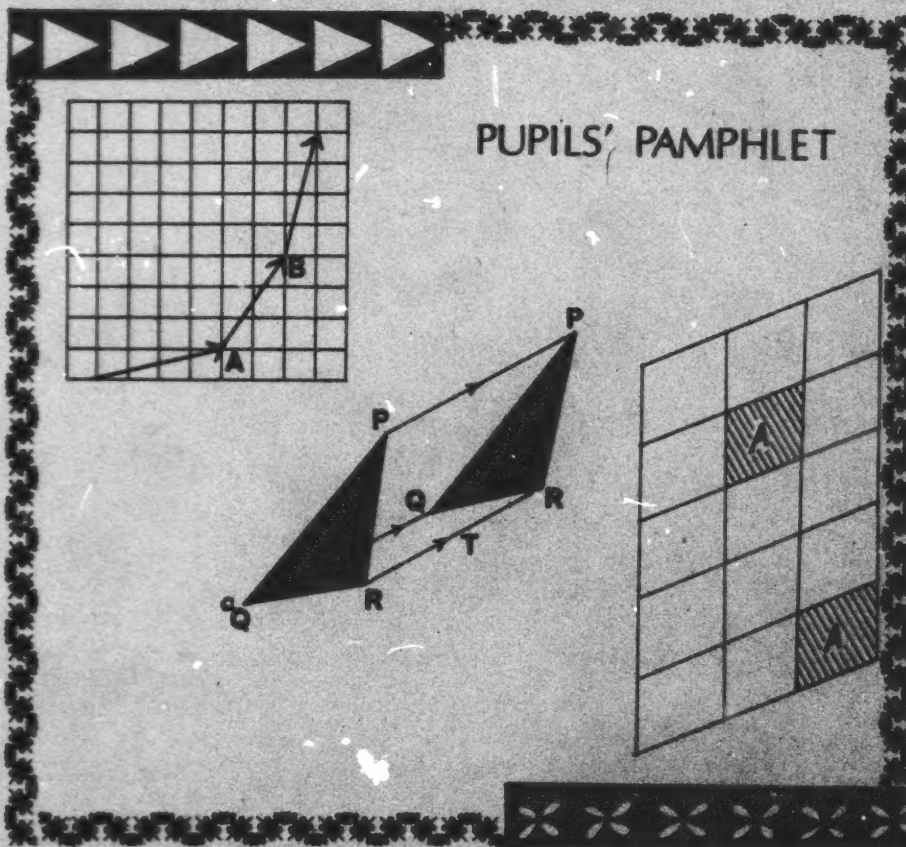
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TRANSFORMATIONS 1

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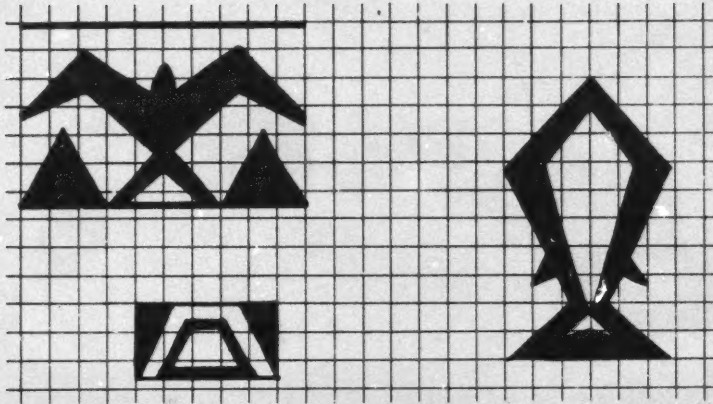
PATTERNS

Many patterns consist of a simple shape repeated.



Study each pattern. Draw the shape that is repeated in the pattern. Write down in your exercise book how it is repeated i.e., is a shape moved to the right, left, up, or down etc. Your teacher will discuss this with you.

Make your own Patterns



Draw the above on squared paper. Cut them out.
Make patterns by moving them and tracing around
the shape.

Your Pattern

Think of a shape. Draw it. Cut it out. Make a
pattern.

Moving Shapes

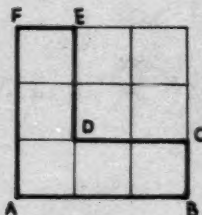
This Pacific Transport bus travels for 50 metres and then stops.



It has changed its position.

- Does it change its shape ?
- Is it still facing in the same direction ?
- Is it still the same size?
- Through what distance does the driver move ?
- Through what distance does the man at the back move ?

An Experiment



- Make a cardboard L shape the same as the one above.
- Draw around the shape and label the points of the shape A B C D E F.
- Place AB on the edge of a ruler.
- Now move the L shape 2 units along the edge of the ruler. Draw around the shape again. Label it

A¹, B¹, C¹, D¹, E¹, F¹.

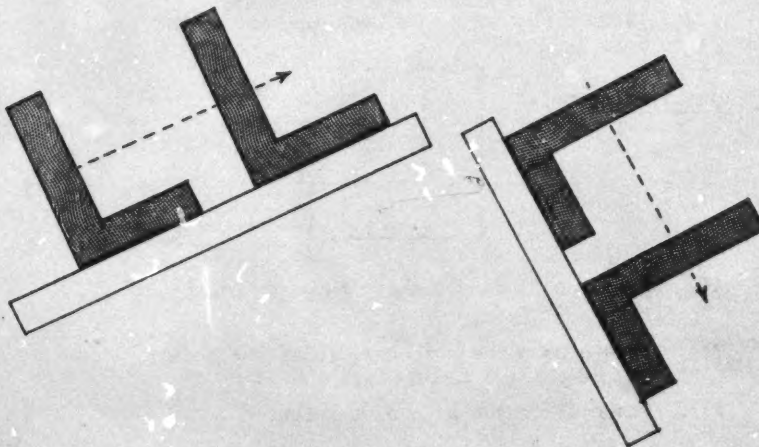
Answer the following questions :

- a) Are the two figures of the same shape ?
- b) Are the two figures of the same size (area) ?

Your teacher will explain to you what "corresponding sides" means.

- c) Are the corresponding sides equal in length ?
- d) Are the corresponding angles equal ?
- e) Are the corresponding sides parallel "
- f) Measure the distance through which each vertex has moved. What do you notice ?

Now move the L shape along the edge of the ruler in different directions. You choose how far you move it.

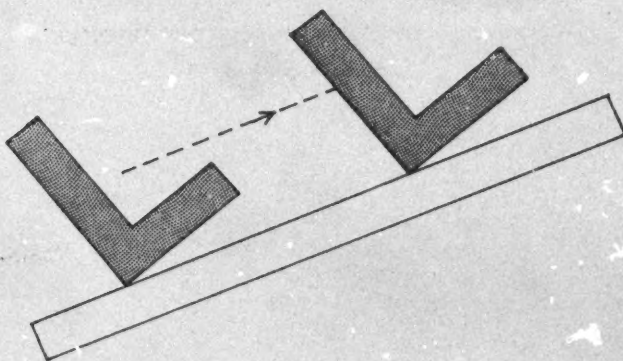


Answer all the questions above each time.

TRANSLATION

The movement of the L shape is called a Translation. When a shape is translated you can imagine it moving along a straight edge. It does not turn as it moves.

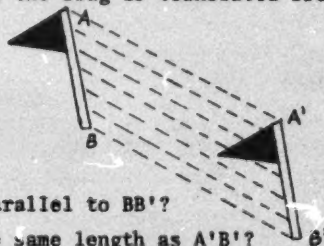
A Problem



The picture show a translation of the L shape. You do this translation. How will you stop the L shape turning ?

Exercise

- 1) In the picture the flag is translated from AB to A'B'



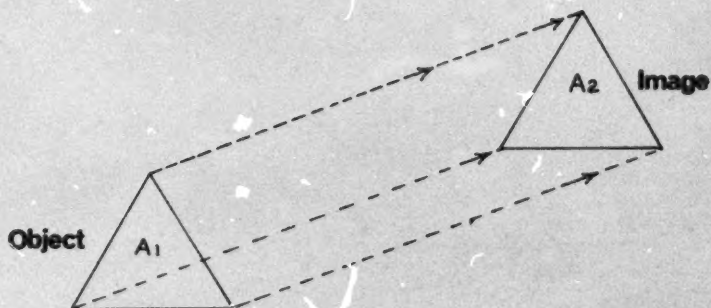
- 1) Is AA' parallel to BB'?
- 2) Is AB the same length as A'B'?
- 3) Is AB parallel to A'B'?
- 4) Could you get A'B' back to AB by a translation?

Summary

A translation of a shape is a movement of a shape:

- 1) The size, area, angles of the shape do not change.
- 2) All points of the shape move along parallel lines.
- 3) Corresponding sides are parallel.

Some New Words



Sometimes we call the first position of the shape the object.

A_1 in the picture is the Object

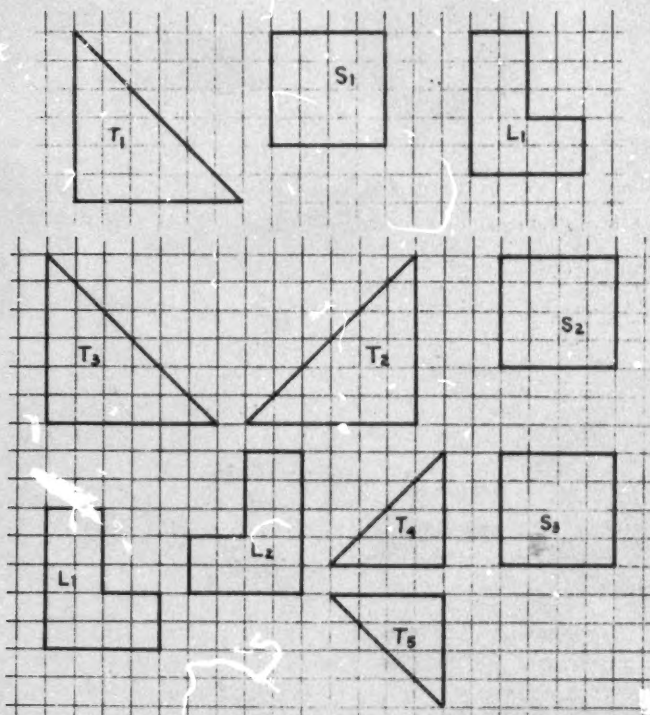
Sometimes we call it the "OBJECT of the translation"

A_2 in the above picture is the "image of the translation".

Are all movements Translations?

Draw and cut out the cardboard shapes T_1 , S_1 , L_1 .

Using your cardboard shapes write down the figures which have been obtained by translation from T_1 , S_1 and L_1 .

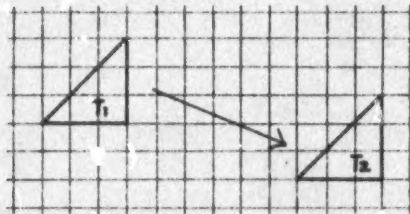


- 1) Is T_2 the same shape as T_1 ?
- 2) Is T_3 the same size as T_1 ?

Discuss with your teacher why you cannot move T_1 to T_2 by a translation.

Write down how you would move a triangle from T_1 to T_2 .

Translating back : Inverse Translation



Look at the picture. T_1 has been moved to T_2 by a translation.

Discussion

- 1) How do you know a translation will get T_2 back to T_1 ?
- 2) What name should we give to the translation that brings T_2 back to T_1 ?

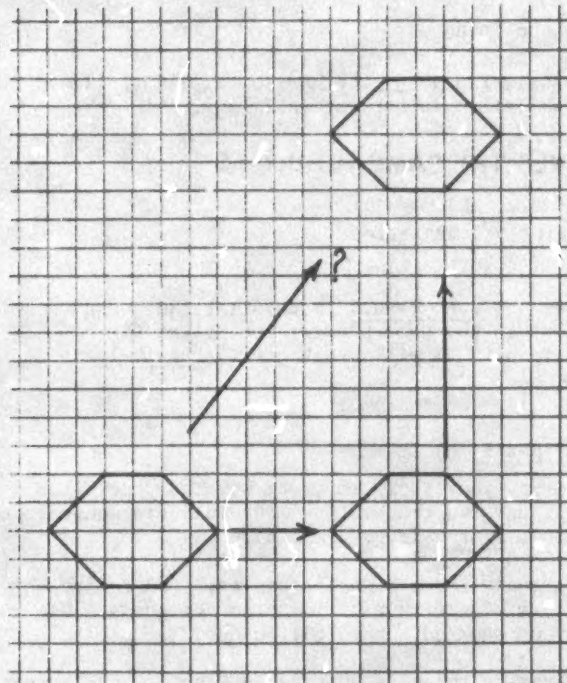
Combinations of Translation

Your teacher will give you a large sheet of squared paper. Use any cardboard shape. You will need a straight edge. (a ruler or a book)

- 1) Start anywhere on the squared paper. Draw around it with pencil. Move the shape 10 units to the right. Draw around it with a pencil.
- 2) Starting from the 2nd position of the shape, move it up 14 units. Draw around the shape.

You have carried out two translations of the shape.

Your squared paper will probably be like the following :



Question

Could you move the shape from its first position to its final position by a single translation? Test it. If you can, describe the translation.

"Equivalent" Translations

You have found out :

A translation along the horizontal line followed by a translation in the vertical line.

GIVES THE SAME RESULT AS

A single translation

"Gives the same result" means that the shape ends up in the same place. For "gives the same result" we will use the word "equivalent"

Experiments

- 1) Using your large sheet of squared paper find if any two translations combined are equivalent to a single transformation.
- 2) Are any three translations combined equivalent to a single translation? Do not guess. Use your squared paper to find out.
- 3) Draw on the squared paper a single translation of any shape in any direction. Now find the combination of horizontal and vertical translations that are equivalent to the single translation.

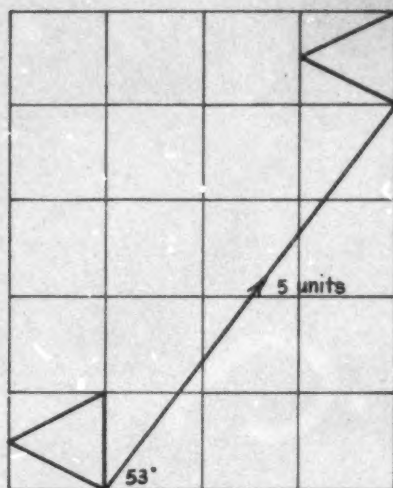
Describing Translations



To Describe a translation means to give all the details so that somebody else could make the translation. Somebody gave this "description" of the above translation.

"The triangle is translated 5 units to the right and upwards."

What is wrong with this description?
Discuss with your teacher some way of correctly describing the translation. There are many ways !



Compare the following two descriptions:

1st description

The triangle is translated 5 units at an angle of 53° North of east

2nd description

The translation of the triangle is equivalent to a horizontal translation of 3 units to the right followed by a vertical translation of 4 units upwards.

- i) Are they both correct descriptions?
- ii) Have they anything in common?
- iii) Which do you prefer? Why?

Mathematical ways of describing translations

We often try to remove words from mathematical sentences. Can you think of examples?

An example :

2 "is the same as" 2 is written

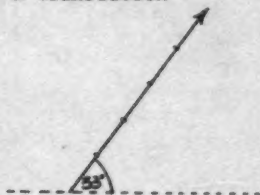
$$2 = 2$$

Look at the ways we replace the following two descriptions by symbols :

The triangle is translated 5 units at an angle of 53° North of East

This is shortened to :

"a translation



The arrow is called an "arrow vector"

- a) Its length shows how big the translation is (in this case 5 units)
- b) Its direction shows the direction of the translation. (in this case 53° north of east.)

The translation of the triangle is equivalent to a horizontal translation of 3 units to the right followed by a vertical translation of 4 units upwards.

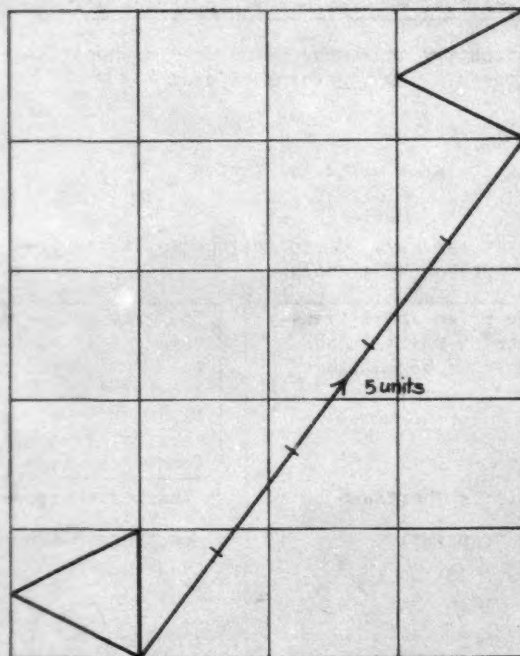
This is shortened to :

"a translation

$$\begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

$\begin{pmatrix} 3 \\ 4 \end{pmatrix}$ is called a vector

- Mathematicians agree that
- a) the 1st number 3 shows a horizontal movement.
- b) the 2nd number 4 shows a vertical movement.

Summary

described
by

a translation

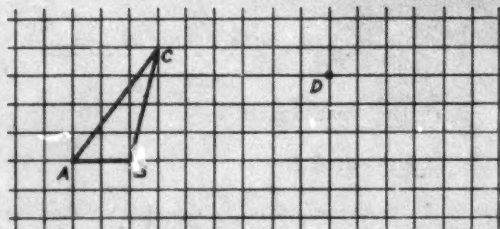


a translation

$$\begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

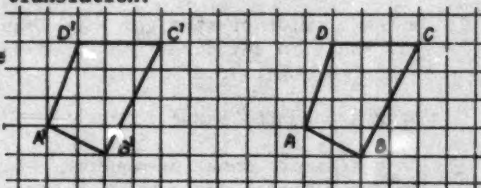
Exercises

1)



A translation moves the triangle ABC so that C moves to D. Copy the diagram and show the image of the translation.

2) Picture



In the diagram $A'B'C'D'$ is the image of $ABCD$ after a translation. In your exercise book (on squared paper) draw the arrow vector of the translation.



We want to move AB to $A'B'$. Explain why no single translation will do this. How do you know that a combination of translation will not move AB to $A'B'$?

3) Cut out a cardboard triangle. Trace around it in any two positions on squared paper.

- Is it always possible to translate the triangle from one position to another?
- There is one shape for which it is always possible to translate it from one position to another.

What is it? Has this anything to do with symmetry?

Use of Arrow Vectors

Combining Translations

An Experiment

- 1) Take any cardboard shape on your large sheet of squared paper: place it anywhere. Draw around it. Call this position 1.

- 2) Using a ruler or straight edge translate the shape in any direction (choose a small number of units).

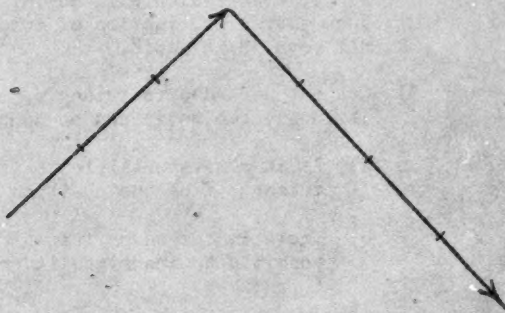
Draw around the shape in position 2.

- 3) On a separate sheet of squared paper draw an arrow vector describing the translation.

Your teacher will help you to do this.

- 4) From position 2 of the shape carry out a second translation. Draw around the shape. Call this the final position.

- 5) At the end of your first arrow vector, draw an arrow vector describing the second translation. You might have something like this



- 6) Now on your squared paper find the single translation equivalent to the first two. Describe it with an arrow vector.
- 7) How could you have got this arrow vector easily from your vector diagram? Your teacher will discuss this with you.

Words

Combining Translations

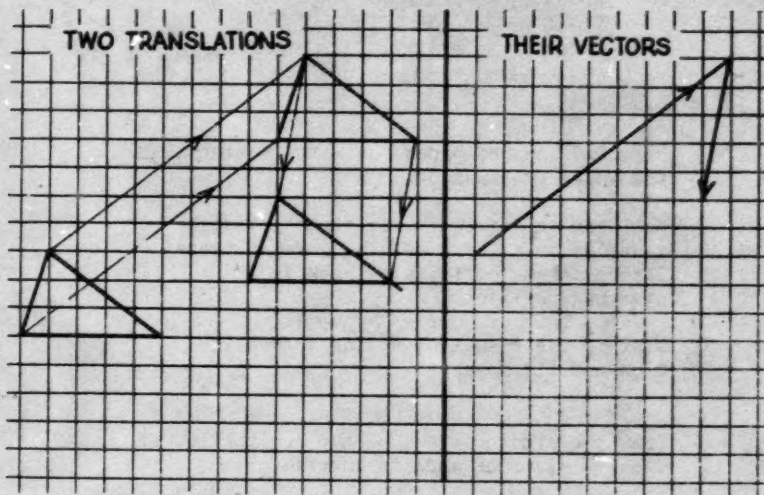
So far we have meant by "combining translation" "doing one translation after another".

Sometimes we will say "adding translations" for "combining translations".

A Problem for Discussion

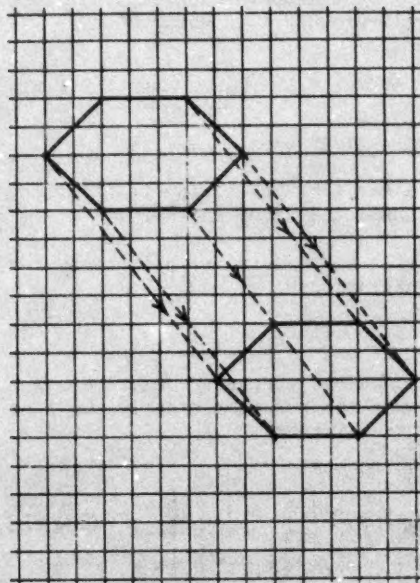
When you draw an arrow vector to describe a translation, must the arrow be exactly the same length as the translation?

Can it be drawn to scale ?



The Inverse Translation and its arrow vector

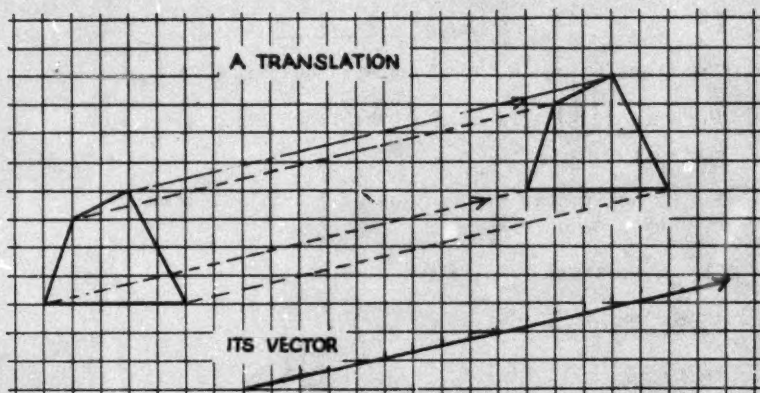
A Problem



- 1) Draw on squared paper the arrow vector of the above translation.
- 2) Now draw the arrow vector of the "inverse translation". Remember the inverse translation takes the triangle back to its starting position.
- 3) What is the result of adding a translation and its inverse?
- 4) Draw the arrow vectors of the combination of the translations and its inverse.

SUMMARY OF ARROW VECTORS

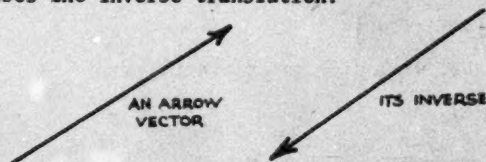
- 1) An arrow vector describe a translation.
 - a) The direction of the arrow vector tells us the direction of the translation.
 - b) The length of the arrow vector tells us the size or length of the translation.

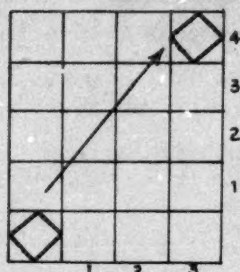


- 2) Use of arrow vectors
 - a) They help us easily to get the translation equivalent to a combination of translations.
 - b) We can also make up combinations of vectors equivalent to a single vector.

Inverse Translations

The arrow reversed i.e. pointing the other way describes the inverse translation.

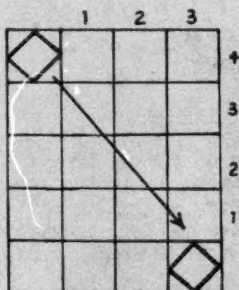


Vectors

The translation in the picture is described by $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$

The vector $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$ tells us that the translation is equivalent to :

- 1) a horizontal translation of 3 units followed by
- 2) a vertical translation of 4 units.

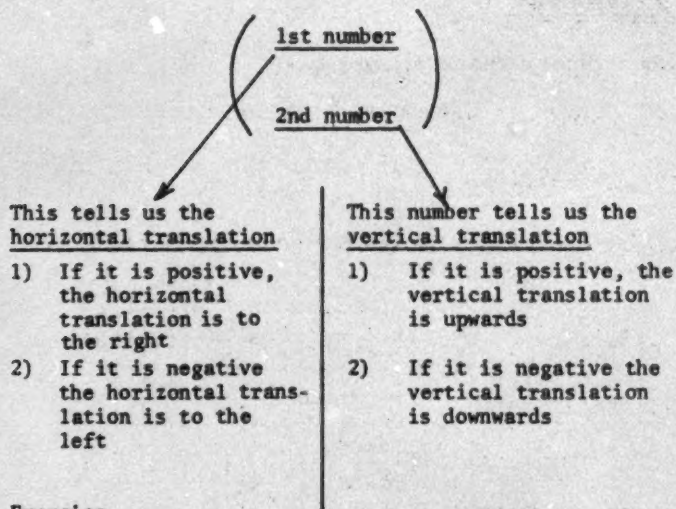
A Problem

What vector will describe the translation in the picture ?

Will $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$ describe it ?

Your teacher will discuss this with you.

You will now have found :



Exercise

On your large sheet of squared paper translate a shape using the following vectors.

$$\begin{pmatrix} 5 \\ 12 \end{pmatrix} \quad \begin{pmatrix} 12 \\ 5 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \begin{pmatrix} 0 \\ 4 \end{pmatrix} \quad \begin{pmatrix} 4 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -5 \\ 12 \end{pmatrix} \quad \begin{pmatrix} 12 \\ -5 \end{pmatrix} \quad \begin{pmatrix} -1 \\ -1 \end{pmatrix} \quad \begin{pmatrix} 0 \\ -4 \end{pmatrix} \quad \begin{pmatrix} -4 \\ 0 \end{pmatrix}$$

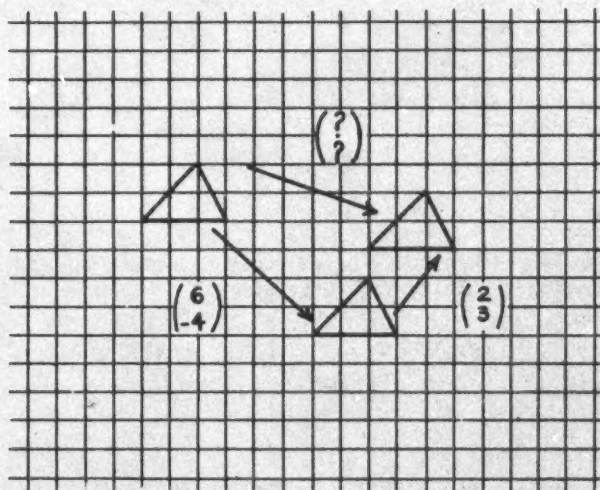
Adding Vectors

An Experiment

On a large sheet of squared paper

- 1) Translate a shape by $\begin{pmatrix} 6 \\ -4 \end{pmatrix}$ followed by
- 2) a translation of $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$

Write down the vector that describes the single equivalent translation



You will have written down $\begin{pmatrix} 8 \\ -1 \end{pmatrix}$

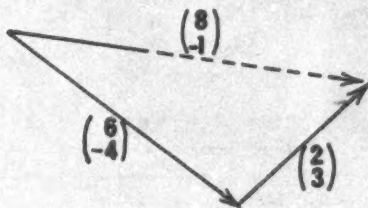
Look at the three vectors $\begin{pmatrix} 6 \\ 4 \end{pmatrix}$ $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ $\begin{pmatrix} 8 \\ -1 \end{pmatrix}$

Can you see how $\begin{pmatrix} 8 \\ -1 \end{pmatrix}$ is got from the others?

You will have found :

When we combine translations, the result is described by 'adding' their vectors.

$$\begin{pmatrix} 6 \\ -4 \end{pmatrix} + \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 8 \\ -1 \end{pmatrix}$$



An Experiment

Take any combinations of the following translations. Translate a shape on squared paper through these combinations. Show that the result is got each time by adding the vectors

$$\begin{pmatrix} 3 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 0 \\ 5 \end{pmatrix} \quad \begin{pmatrix} -6 \\ 7 \end{pmatrix} \quad \begin{pmatrix} -7 \\ 6 \end{pmatrix} \quad \begin{pmatrix} -4 \\ 2 \end{pmatrix} \quad \begin{pmatrix} 5 \\ 0 \end{pmatrix}$$

Answer these questions each time :

- 1) Does the size and direction of the translation depend on where you start in the sheet ?
- 2) Suppose you were to draw an x and y axis somewhere on your paper. Would it make any difference to the translation? Your teacher will discuss this last question with you.
- 3) Write down the vector that describes the inverse translation of each of the translations above.

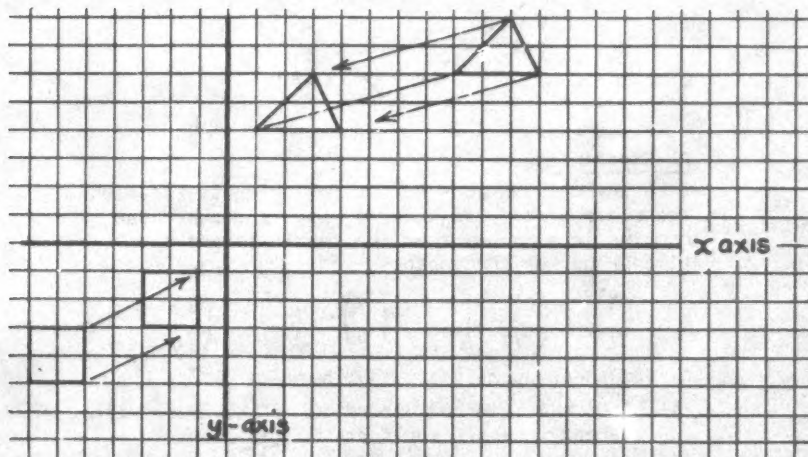
(Do the inverse translation on squared paper: then describe it.)

Combining a Translation and its Inverse

On squared paper move a shape

- 1) by a translation $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$
- 2) Now, move it $\begin{pmatrix} -3 \\ -4 \end{pmatrix}$
- 3) What is the result?
- 4) What is $\begin{pmatrix} 3 \\ 4 \end{pmatrix} + \begin{pmatrix} -3 \\ -4 \end{pmatrix} = ?$

A Problem

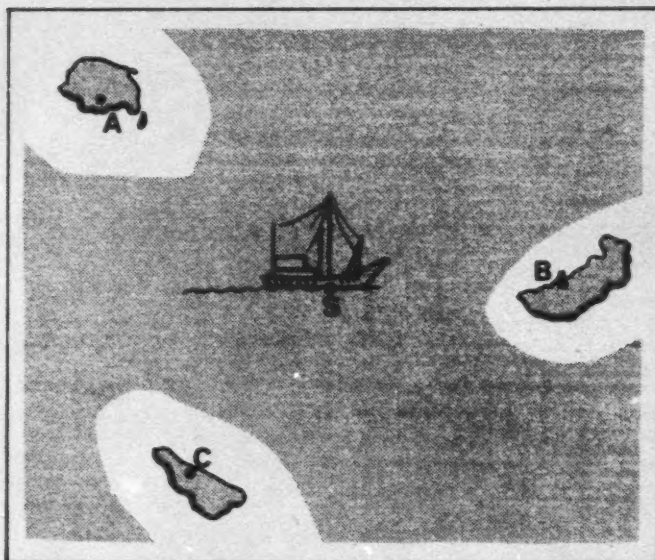


Write down a vector that describes

- 1) The translation of the square.
- 2) The translation of the triangle.
- 3) If the x and y axis were placed somewhere else, would your answers be different ?

Another use for Vectors

We have used arrow vectors, and vectors to describe translations. They have other uses.



A ship is at the place marked S. There is a village on each of the islands at A,B,C.

at A they ask:

"where is the ship?"

at B they ask:

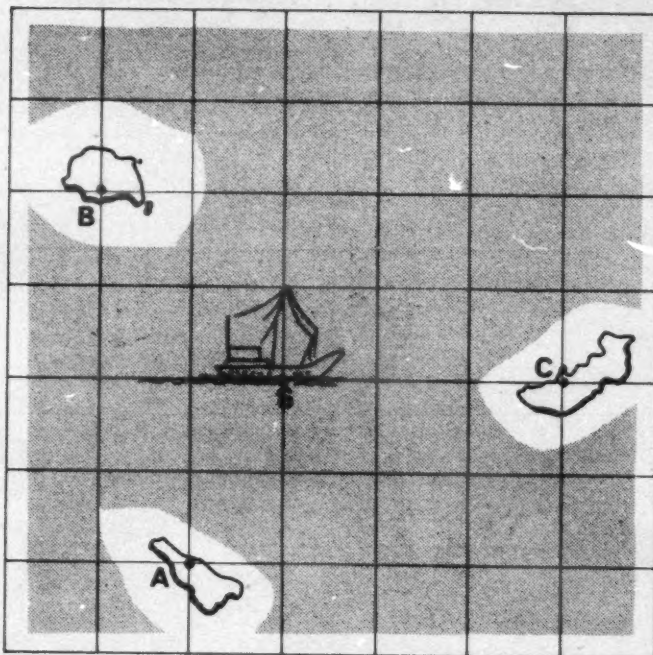
"where is the ship?"

at C they ask:

"where is the ship?"

How would you answer these questions ?

Here is one way used



A is told

The ship is at

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

B is told

The ship is at

$$\begin{pmatrix} 2 \\ -2 \end{pmatrix}$$

C is told

The ship is at

$$\begin{pmatrix} -3 \\ 0 \end{pmatrix}$$

- $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ is called the position vector of S relative to A
 $\begin{pmatrix} 2 \\ -2 \end{pmatrix}$ is called the positive vector of S relative to B
 $\begin{pmatrix} -3 \\ 0 \end{pmatrix}$ is called the positive vector of S relative to C.

"relative to A" means "from A".

When A asked :

"where is the ship ?"

He was perhaps more interested in

- 1) how far the ship was from A
- 2) in what direction the ship lay from A

$\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ answers these questions.

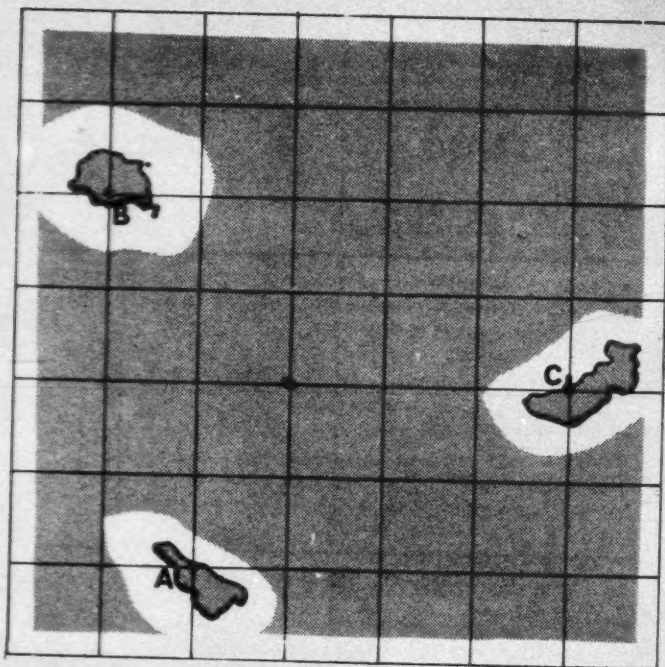
Can you see any disadvantages in the "position vector" way of saying where things are

Discuss this with your teacher.



Suppose you are the captain of a ship. You are not sure of where your ship is. Which is the more useful information to you :

- 1) There is a reef at (lat. 5° , long. 159°)
- 2) There is a reef at $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ miles



Look at the picture :

- 1) What is the position vector of B relative to A ?
- 2) What is the position vector of C relative to B ?
- 3) From your last two answers how would you get the position vector of C relative to A ?
- 4) What is the position vector of A relative to B ? Compare this with your answer to (1) above. What do you find ?
- 5) Look at your answer to (2). Now, from this, write down the position vector of B relative to C?

Translations and Operators

We often use letter operators to stand for a translations.

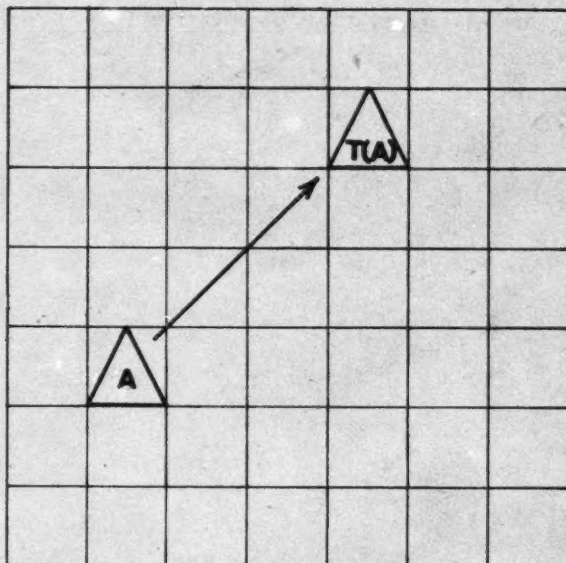
Example

Let T be the translation $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$

This picture shows

- 1) a triangle called A
- 2) $T(A)$

Your teacher will remind you what $T(A)$ means



Write down what T^{-1} is

Let S be the translation $\begin{pmatrix} 4 \\ -5 \end{pmatrix}$

Now draw a triangle on squared paper. Call it A

- a) Draw
- (1) $A(A)$
 - (2) $ST(A)$
 - (3) $TS(A)$
 - (4) $S^{-1}T(A)$ First write down what S^{-1} is

b) What is $SS^{-1}(A)$?

c) Is $TS(A) = ST(A)$?

d) Are all translations commutative ?